

A 4-node geometrically nonlinear degenerated shell element for **fabric** modeling

This work is aimed at creating FEM-like fast methods for calculating deformations (wrinkling/draping) for clothing worn by soldiers in Nuclear, Biological, and Chemical (NBC) warfare.

By:

Salam Rahmatalla

Virtual Soldier Research
The University of Iowa

<http://www.engineering.uiowa.edu/~amalek/VSR>

<http://www.digital-humans.org>



Objectives

- Formulate a geometrical nonlinear thin shell element for fabric modeling.
- Test the model to simulate drape over complex arbitrary surfaces.
- Test the model to capture buckling and wrinkles.
- Extend the model to analyze fabrics with seams, stitches and multiple ply fabric assemblies.
- Extend the model to include contact with surfaces.



Degenerated shell element

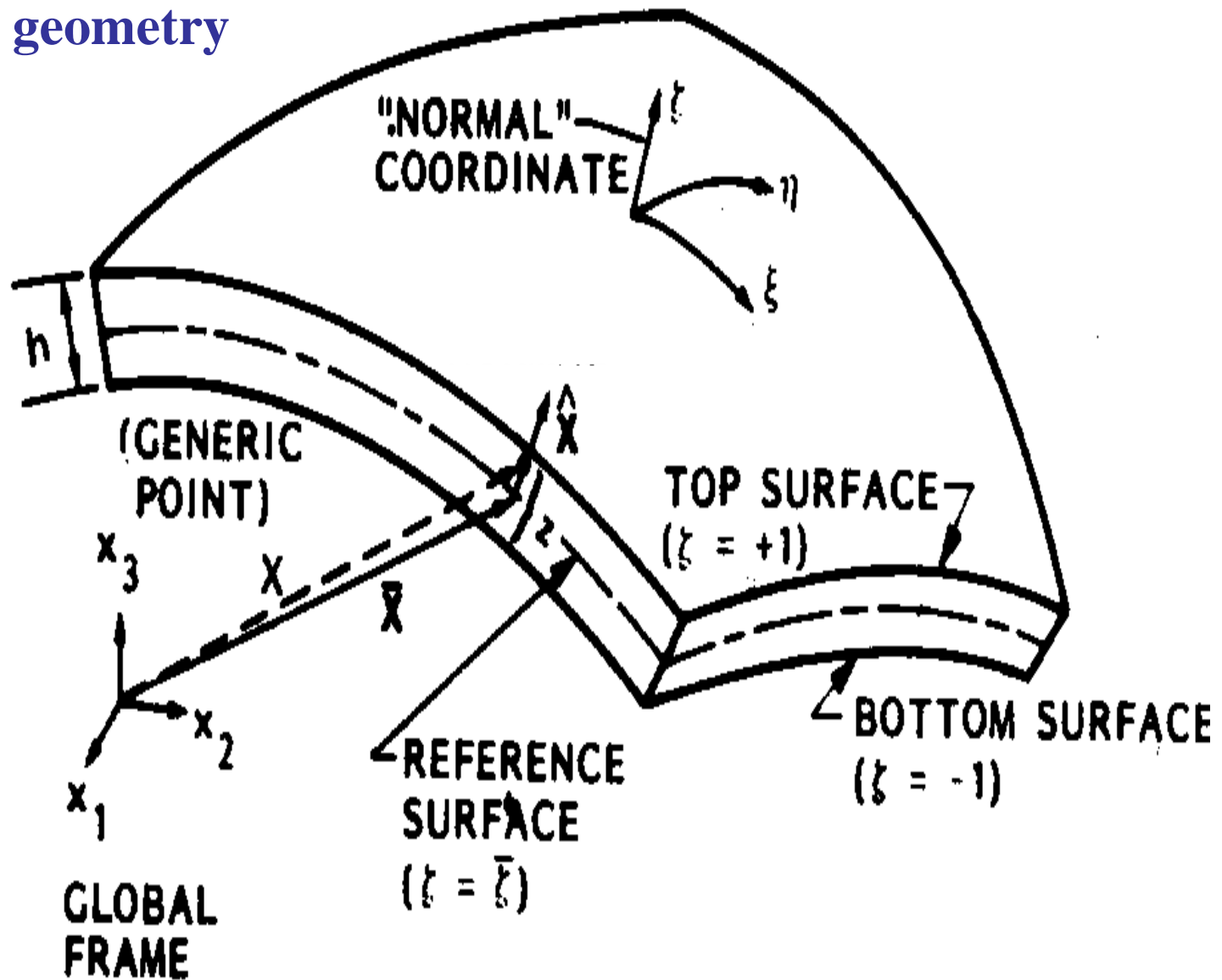
Assumptions are:

1. Fibers remain straight (modified Mindlin-Reissner assumption)
2. The stress normal to the midsurface vanishes (also called the plane stress condition)

Note: Selective reduced integration is used to avoid membrane and transverse shear locking.



Shell geometry



Shell geometry

$$\mathbf{x}(\mathbf{x}, \mathbf{h}, \mathbf{z}) = \bar{\mathbf{x}}(\mathbf{x}, \mathbf{h}) + X(\mathbf{x}, \mathbf{h}, \mathbf{z})$$

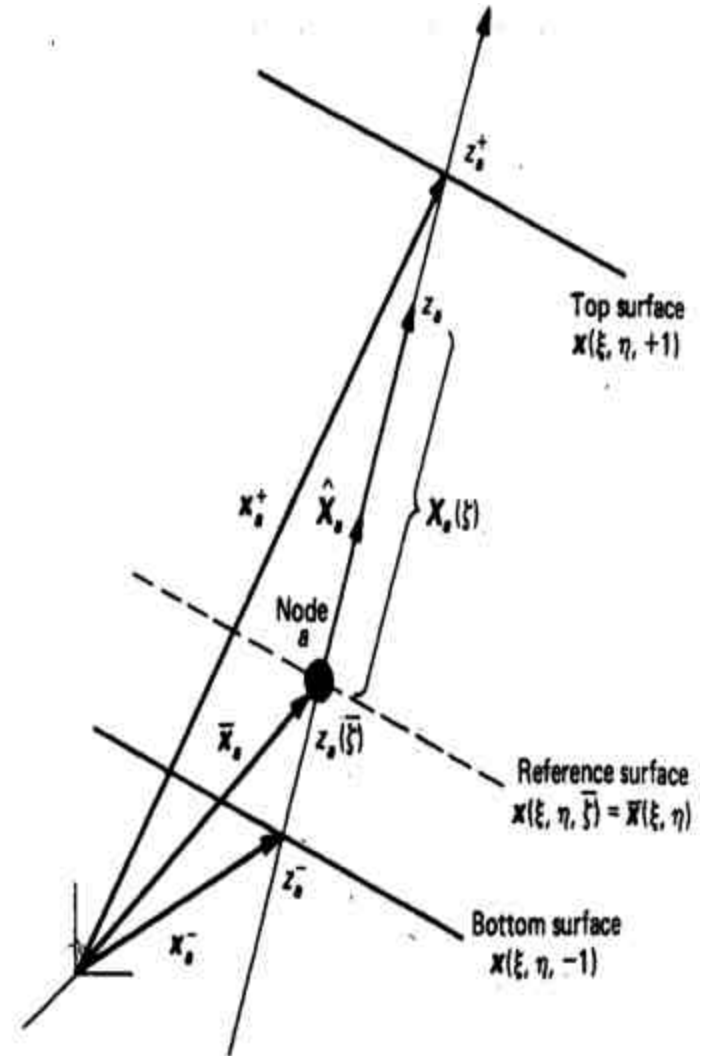
$$\bar{\mathbf{x}}(\mathbf{x}, \mathbf{h}) = \sum_{a=1}^{nen} N_a(\mathbf{x}, \mathbf{h}) \bar{\mathbf{x}}_a$$

$$X(\mathbf{x}, \mathbf{h}, \mathbf{z}) = \sum_{a=1}^{nen} N_a(\mathbf{x}, \mathbf{h}) X_a(\mathbf{z})$$

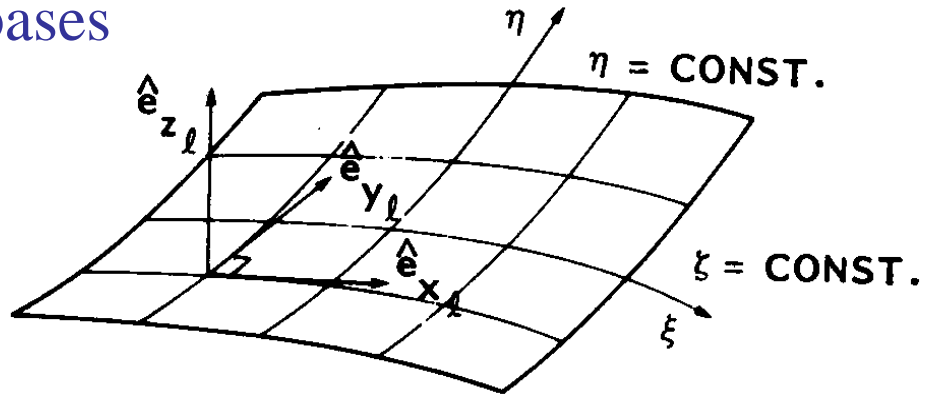
$$X_a(\mathbf{z}) = z_a(\mathbf{z}) \hat{X}_a$$

$$z_a(\mathbf{z}) = N_+(\mathbf{z}) z_a^+ + N_-(\mathbf{z}) z_a^-$$

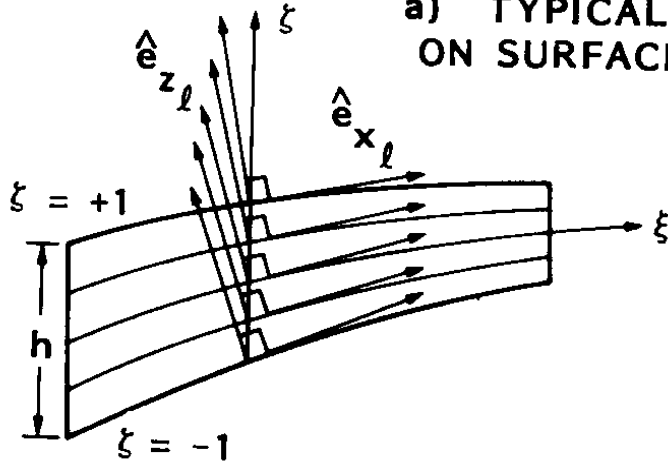
$$N_-(\mathbf{z}) = \frac{1}{2}(1 - \mathbf{z}) \quad N_+(\mathbf{z}) = \frac{1}{2}(1 + \mathbf{z})$$



Lamina and fiber bases

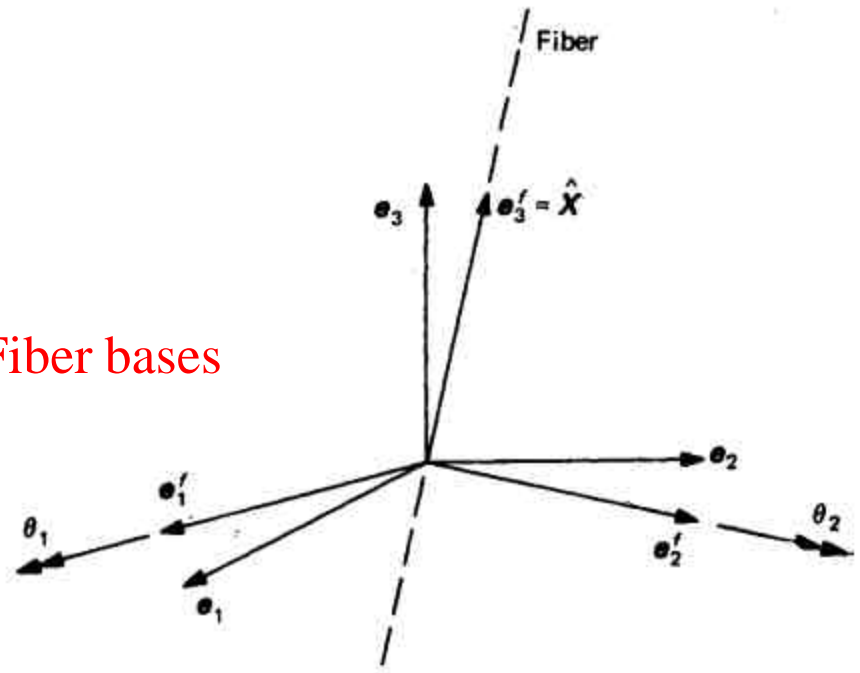


a) TYPICAL LAMINA BASIS ON SURFACE: $\zeta = \text{CONST.}$



Lamina basis

Fiber bases



Shell kinematics

$$\mathbf{u}(\mathbf{x}, \mathbf{h}, \mathbf{z}) = \sum_{a=1}^{nen} N_a(\mathbf{x}, \mathbf{h}) \mathbf{u}_a + \sum_{a=1}^{nen} N_a(\mathbf{x}, \mathbf{h}) z_a (\sin \mathbf{q}_{a2} \mathbf{e}_{a1}^f - \sin \mathbf{q}_{a1} \mathbf{e}_{a2}^f)$$

$$u_{i,I}^l = \sum_{m=1}^3 q_{im} \sum_{a=1}^{nen} \left\langle N_{a,I} u_{am} + (N_a z_a)_{,I} (\sin \mathbf{q}_{a2} e_{am1}^f - \sin \mathbf{q}_{a1} e_{am2}^f) \right\rangle$$

$$d\mathbf{E} = \begin{pmatrix} F_{i1} d\mathbf{u}_{i,1} \\ F_{i2} d\mathbf{u}_{i,2} \\ F_{i3} d\mathbf{u}_{i,3} \\ F_{i1} d\mathbf{u}_{i,2} + F_{i2} d\mathbf{u}_{i,1} \\ F_{i2} d\mathbf{u}_{i,3} + F_{i3} d\mathbf{u}_{i,2} \\ F_{i3} d\mathbf{u}_{i,1} + F_{i1} d\mathbf{u}_{i,3} \end{pmatrix} ; \quad d\mathbf{E}_{(5 \times 1)} = \sum_{a=1}^{nen} \mathbf{B}_{(5 \times 5)}^a \begin{pmatrix} d\bar{\mathbf{u}}_a \\ dq_{a1} \\ dq_{a2} \end{pmatrix}_{(5 \times 1)}$$



Hyperelastic material model

The particular strain energy function used here is that of Ciarlet, wherein the volumetric (U) and deviatoric (\bar{W}) strain energy functions are assumed to be decoupled and of the forms:

$$E(\mathbf{F}) = U(J) + \bar{W}(\bar{\mathbf{C}})$$

$$U(J) = \frac{1}{2} K \left[\frac{1}{2} (J^2 - 1) - \ln(J) \right]$$

$$\bar{W} = \frac{1}{2} \mathbf{m} [tr(\bar{\mathbf{C}}) - 3]$$



Stiffness, stress, and elasticity tensor expressions

From the strain energy function, we can find expressions for the stress tensor, elasticity tensor, and the stiffness matrix .

$$\mathbf{S} = \frac{k}{2}(J^2 - 1)\mathbf{C}^{-1} + \mathbf{m}J^{(-\frac{2}{3})}\mathbf{1} - \frac{1}{3}\mathbf{m}J^{(-\frac{2}{3})}tr(\mathbf{C})\mathbf{C}^{-1}$$

$$C_{IJKL} = \left\{ KJ^2 + \frac{2}{9}\mathbf{m}J^{(-\frac{2}{3})}tr(\mathbf{C}) \right\} C_{IJ}^{-1}C_{KL}^{-1} - \left\{ K(J^2 - 1) - \frac{2}{3}\mathbf{m}J^{(-\frac{2}{3})}tr(\mathbf{C}) \right\} C_{IK}^{-1}C_{JL}^{-1} \\ - \frac{2}{3}\mathbf{m}J^{(-\frac{2}{3})} (C_{KL}^{-1}\mathbf{d}_{IJ} + \mathbf{d}_{KL}C_{IJ}^{-1})$$

$$\mathbf{K}_{IL}^{AB} = \int_{\Omega_S} B_{JI}^A C_{JK} B_{KL}^B d\Omega_S + \int_{\Omega_S} N_{,J}^A S_{IJ} N_{,K}^B \mathbf{d}_{IL} d\Omega_S$$



References

Hughes, T.J.R, (2000), The Finite Element Method, Mineola, New York.

Zienkiewicz, O.C., and Taylor, R.L., (2000), The Finite Element Method,
Volume 2: Solid Mechanics. Butterworth Heinemann, Oxford.

FENDAC software, C.C.Swan.



Contact Information:

Virtual Soldier Research program

The University of Iowa

Tel. 319-335-5722

Email: deborah-hampton@uiowa.edu

<http://www.engineering.uiowa.edu/~amalek/VSR>

<http://www.digital-humans.org>

